Optimal Interconnect Diagnosis of Wiring Networks

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Abstract— **Interconnect diagnosis is an important problem in very large scale integration (VLSI), multi-chip module (MCM) and printed circuit board (PCB) production. The problem is to detect and locate all the shorts, opens and stuck-at faults among a set of nets using the minimum number of parallel tests. In this paper, we present worst-case optimal algorithms and lower bounds to several open problems in interconnect diagnosis.**

Keywords— **Diagnosis, Test, Interconnect.**

I. Introduction

A WIRING network consists of a set of nets $W = \{w_1, w_2, \ldots, w_n\}$. A net contains one or more interconnected drivers and one or more receivers. The logic value of a fault-free net is controlled by its drivers and observed by all of its receivers. There are three types of faults considered: shorts, opens and stucks (stuck-at-0 and stuck-at-1). If nets $w_1, w_2, \ldots, w_k, k \geq 2$, are shorted together, and the logic values of the drivers controlling the k nets are $x_1, x_2, \ldots, x_k$ respectively, then the receivers on these nets will receive $x_1 \vee x_2 \vee \cdots \vee x_k$. This is the *wired-OR* model. (In the wired-AND model, the short causes the nets to receive $x_1 \wedge \cdots \wedge x_k$, which is the dual of the wired-OR model. The results for the wired-OR model in this paper can be easily extended to the other model.) If an open fault separates a set of drivers and receivers into two disjoint parts, then each part will behave like an independent net. A wire not connected to any driver is assumed to float to a logic value 0 or 1. This value is assumed not to change during the diagnosis process. A net shorted to the power or ground is stuck-at-1 or stuck-at-0 respectively.

Interconnect diagnosis has many applications, including built-in self-test, boundary scan, and design for test of VLSI, MCMs, and PCBs [2], [4], [6], [7], [8], [11], [13], [16], [17]. In this paper, we study behavioral diagnosis, that is the detection and location of all single and multiple faults without utilizing any information regarding the particular routing of the internal connections or the physical failure mechanism. The basic test operation is the *parallel test*, that is, 0's and 1's are sent from the drivers and outputs are observed from the receivers. The objective is to identify the faults using the minimum number of parallel tests.

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Lien and Breuer [11] defined seven levels of *diagnostic resolution.* Since some of their classification is based on procedural restrictions rather than on the final result of the diagnosis, we define a comparable, yet simpler form of diagnostic resolution (DR):

- DR 1: Determine if there is any fault.
- DR 2: Identify all the nets that involve faults.
- DR 3: Identify all the connectivity.

DR1 is also called fault detection, and DR3 is also called full fault diagnosis. It is clear that DR1 is easier than DR2, which in turn is easier than DR3.

From the output of each test, all we can learn is the information about the connectivity between drivers and receivers. Therefore, the diagnosis can only tell us the connectivity between drivers and receivers, and its transitive closure. Examples of faults that cannot be diagnosed include multiple shorts between two nets, multiple opens within a net, shorts among a set of receivers that are not connected to any drivers, and shorts among a set of drivers that are not connected to any receiver.

Previous work on interconnect diagnosis has primarily been based on one of three fault models:

- Shorts only.
- Shorts and stucks.
- Shorts, stucks and opens.

Notice the fault model for shorts and stucks is similar to the fault model of shorts with two additional nets: the power and the ground. Therefore in this paper, we focus on the interconnect diagnosis for shorts only, and for shorts, opens and stucks.

There are variations in the manner the diagnosis can be performed [4], [6], [7], [8], [9], [11], [16].

In a *self-diagnosis*, the result of diagnosing each net must be decided only by looking at the output of that net. In a *global diagnosis*, the result can be decided by combining the information gathered from all the nets.

In an *adaptive* diagnosis, the input of each test is based on the output of previous tests. In a *non-adaptive* diagnosis, all test inputs are decided before the application of any test. It is not hard to see that for adaptive diagnosis, we must use the global information to decide the next test.

A diagnosis algorithm is *optimal* if for all n, the algorithm uses the minimum number of parallel tests to diagnose networks of n nets, among all diagnosis algorithms. In addition, the computational time for test generation and test result analysis must be a polynomial in terms of n.

II. Review of Previous Results

Table I summarizes the previous best results of interconnect diagnosis for shorts only, and Table II summarizes the previous best results for opens, shorts and stucks. In the tables, question marks represent open problems, all logs

TABLE I
Previous best diagnosis algorithms for shorts.

Diagnostic Resolution	Adaptive	Global	Number of Tests	Optimal	Authors
1	no	no	$\lceil \log n \rceil$	yes	Kautz [9]
2	no	no	m	yes	Cheng, Lewandowski and Wu [4]
	no	yes	?	?	
	yes	yes	?	?	
3	no	no	n	yes	Hassan, Rajski and Agarwal [7]
	no	yes	?	?	
	yes	yes	$\lceil \log n \rceil + k$	?	Jarwala and Yau [8]

TABLE II
Previous best algorithms for shorts, opens and stucks.

Diagnostic Resolution	Adaptive	Number of Tests	Optimal	Authors
1	no	$\lceil \log(n+2) \rceil$?	?	Goel and McMahon [6]
2	no	m	yes	Cheng, Lewandowski and Wu [4]
	yes	?	?	
3	no	$n+1$	yes	Lien and Breuer [11]
	yes	$m+k$	?	Lien and Breuer [11]

are base 2, n is the number of nets, k is the number of faulty nets, and

$$m = \min\{i : i \text{ an integer and } \binom{i}{\lceil \frac{i}{2} \rceil} \geq n\} \qquad (1)$$

which is $\log n + \frac{1}{2}\log\log n + O(1)$. In this paper, we solve most of the open problems listed in Table I and Table II.

For diagnosis of shorts only, Kautz [9] showed that $\lceil \log n \rceil$ tests are necessary and sufficient to detect if there is any short in a set of n nets. The idea is to apply a unique input bit string to each net. If there is no short between any two nets, then the output bit string should be identical to the input bit string for each net. A short will increase the number of 1's for some nets, hence the fault will be detected. The so called *counting sequence* method is optimal for DR1, i.e., to detect if the network contains any short.

However, Kautz's method does not identify all the nets involved in shorts. Wagner [16] improved the diagnostic resolution by applying Kautz's method twice, once in the original form and once in its complement. This way, all nets involved in shorts receive more 1's than the counting sequence or the complement of the counting sequence. Cheng, Lewandowski and Wu [4] used the *maximum independent set* method to reduce the number of tests to m, see formula (1). A maximum independent set is a set of sequences that contains an equal number of 1's and 0's. From Sperner's Theorem [1], the maximum independent set method is optimal for non-adaptive, self-diagnosis with DR2, i.e., to identify all the nets that involve shorts. They raised the question of the optimality of their algorithm for global diagnosis [4].

To reach DR3, i.e., to identify all the connectivity, Hassan, Rajski and Agarwal [7] proposed the *walking one's* method for non-adaptive self-diagnosis. In the walking one's method, only one net is assigned to 1 at each test. After n tests, we can look at the output at each net w_i to tell which nets are shorted to w_i. The optimality of the walking one's method is obvious because for each w_i, we need $n-1$ bits to represent all possible shorts with other nets, and w_i has to send 1 at least once. To adaptively diagnose the faults, Jarwala and Yau [8] proposed a two-step algorithm that consists of a counting sequence followed by a walking one's sequence for all the faulty nets identified by the counting sequence. Let k be the number of faulty nets, then it is easy to see the counting sequence will detect at least $\lceil k/2 \rceil$ and at most k faulty nets. Therefore in the worst case, the algorithm of Jarwala and Yau uses $\lceil \log n \rceil + k$ tests, and k could be of order n.

For networks with opens, shorts and stucks, Goel and McMahon [6] proposed the *modified counting sequence* method for detecting if the network contains any shorts and stucks. The modified counting sequence method is similar to the counting sequence method, except the bit strings $00\cdots0$ and $11\cdots1$ are not used. They did not consider opens explicitly since they treat opens as stuck-at-1. However, opens can be combined with shorts to form complicated connections, and therefore may not be stuck at any value. The maximum independent set method [4] can be used to non-adaptively identify all faulty nets of opens, shorts and stucks. Lien and Breuer [11] defined maximal diagnosis which is equivalent to our DR3. They proposed walking one's for non-adaptive diagnosis of DR3 and proved it is optimal. They also gave a two-step adaptive diagnosis that uses $m + k$ tests, where m is defined in formula (1) and k is the number of faulty nets.

Finally, we review other related research. Interconnect diagnosis based on structural information has been studied by Garey, Johnson and So [5], Cheng, Lewandowski and Wu [4], Lien and Breuer [11], and McBean and Moore [13]. Yau and Jarwala [17] studied the tradeoff between the number of tests and the fault coverage. Chan [2] combined the walking one's sequence with signature analysis, thereby making the diagnosis more attractive for boundary scan test. Chen and Hwang [3] used a different model where all receiving nets are tied together at each test, which is known as group diagnosis.

III. Diagnosis of Shorts

In this section, we assume there are only shorts in the network. This is the fault model also used by many previous researchers, see Table I. Since there are no opens, we assume without loss of generality there is only one driver and one receiver for each net.

A. Adaptive Global Diagnosis

It is known that $\lceil \log n \rceil$ tests are necessary to detect whether the network contains any shorts [9]. Now we show $\lceil \log n \rceil$ tests are also sufficient to identify all the shorted nets and their connected components.

For a set of nets S, we call R, $R \subseteq S$, the set of *representatives* of S if R can reach all the nets in S. In other words, if we send 1's from the drivers of R and 0's from the drivers of $S - R$, then all nets in S will receive 1. Our algorithm *AGD* (Adaptive Global Diagnosis) uses the divide-and-conquer strategy on the set of representatives.

Algorithm. $AGD(S, R)$.
Input. S is a set of nets, and $R \subseteq S$ is the set of representatives of S.
Output. All the connected components.

Method.
if $|R| = 1$
then Report S as one connected component
else Arbitrarily pick $R' \subset R$ such that $|R'| = \lceil |R|/2 \rceil$;
Send 1 from R', and 0 from $S - R'$;
Let $S_0 = \{w \in S \mid w \text{ receives } 0\}$ and
$S_1 = \{w \in S \mid w \text{ receives } 1\}$;
Treat S_0 and S_1 as two disjoint networks and recurse in parallel $AGD(S_1, R'), AGD(S_0, R - S_1)$
endif.
End of Algorithm.

We start by invoking the algorithm $AGD(W, W)$.

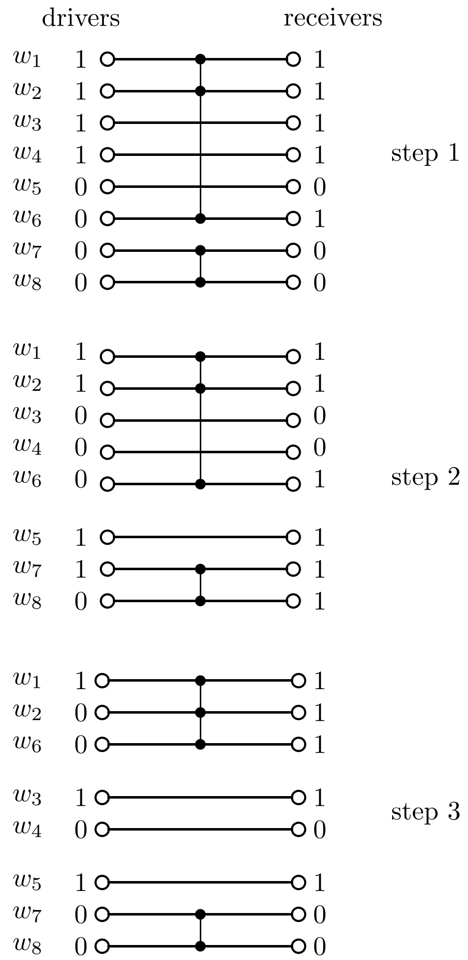

Fig. 1. Example adaptive diagnosis of 8 nets.

The example in Figure 1 illustrates how the algorithm works on a set of 8 nets, where w_1, w_2, w_6 are shorted together, and w_7, w_8 are shorted together. We start with $S = \{w_1, \ldots, w_8\}$ and $R = \{w_1, \ldots, w_8\}$. In step 1, we send 1 from half of the representatives $\{w_1, w_2, w_3, w_4\}$, and 0 from the remaining nets $\{w_5, w_6, w_7, w_8\}$. The outputs received by the receivers are shown in the figure. Since w_6 receives 1, we know it is shorted to the first half of the nets. Also since w_5, w_7 and w_8 receive 0, we know they are disjoint from the first half. Therefore we can partition the network into two disjoint sub-networks $\{w_1, w_2, w_3, w_4, w_6\}$ and $\{w_5, w_7, w_8\}$, and recurse on each sub-network independently. In step 2, for the sub-network of $\{w_1, w_2, w_3, w_4, w_6\}$, the set of representatives is $\{w_1, w_2, w_3, w_4\}$. Therefore we sent 1 from $\{w_1, w_2\}$ and 0 from $\{w_3, w_4, w_6\}$. For the sub-network $\{w_5, w_7, w_8\}$, the set of representatives is $\{w_5, w_7, w_8\}$. Therefore we send 1 from $\{w_5, w_7\}$ and 0 from $\{w_8\}$. The output of step 2 further partitions the nets into three sub-networks: $\{w_1, w_2, w_6\}$, $\{w_3, w_4\}$ and $\{w_5, w_7, w_8\}$, with their sets of representatives being $\{w_1, w_2\}$, $\{w_3, w_4\}$, and $\{w_5, w_7\}$ respectively. Therefore in step 3, we send 1 from $\{w_1\}$, $\{w_3\}$, $\{w_5\}$, and 0 from the remaining nets.

Theorem 1: Algorithm AGD correctly finds all connected components in total $\lceil \log n \rceil$ tests, where n is the number of nets.

Proof: We first prove by induction on the size of R, that the algorithm correctly finds all connected components.

When $|R| = 1$, by the definition of R, all nets in S are connected to R. Therefore the algorithm is correct to report S as one connected component.

When $|R| > 1$, the algorithm performs one test and splits S into S_0 and S_1. Clearly, there cannot be any shorts between S_0 and S_1. It is also clear from the way we perform the test that R' is the set of representatives for S_1. Since R is the set of representatives for S, $R - S_1$ must be the set of representatives for S_0. Since $|R'| = \lceil |R|/2 \rceil < |R|$ and $|R - S_1| < |R - R'| = \lfloor |R|/2 \rfloor < |R|$, from the induction hypothesis, $AGD(S_1, R')$ and $AGD(S_0, R - S_1)$ will correctly diagnose the disjoint sub-networks respectively. Therefore, the algorithm works correctly.

Now consider $T(i, j)$, the number of parallel tests used by the algorithm to diagnose a set of nets S with the set of representatives R, where $|S| = i$ and $|R| = j$.

$$T(i,j) \leq \begin{cases} 0 & \text{if } j = 1, \\ 1 + T(i, \lceil \frac{j}{2} \rceil) & \text{if } j > 1. \end{cases}$$

Solving the equation we have $T(n, n) \leq \lceil \log n \rceil$. ■

B. Non-Adaptive Global Diagnosis of DR2

To non-adaptively diagnose a set W of n nets in t tests, we apply a *test pattern* $\mathcal{T} = \{X_1, X_2, \ldots, X_n\}$ of *size* n and *length* t to W and observe the output. Each $X_i = x_{i1}x_{i2}\cdots x_{it}$ is a t-bit binary string and is called a *sequential test vector*. At step j of the diagnosis, net w_i will send bit x_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, t$. For convenience, we also write the net sending the sequential test vector X as $w(X)$. Table III is an example test pattern of length 3 for non-adaptive diagnosis of 5 nets.

Lemma 1: The necessary and sufficient condition for a test pattern $\mathcal{T}$ to reach DR2 (i.e., to identify all faulty nets) for non-adaptive global diagnosis is that there are no distinct $Y, Z_1, \ldots, Z_j \in \mathcal{T}$, for any $j \geq 1$, such that

$$Y = \vee_{i=1}^{j} Z_i.$$

Proof: The necessity is because otherwise if there are $Y = \vee_{i=1}^{j} Z_i$, then we can not distinguish the following two cases, which is also called the *aliasing syndrome* [8]:

- Nets $w(Z_1), \ldots, w(Z_j)$ are shorted together, and all other nets including $w(Y)$ are fault-free.
- Nets $w(Z_1), \ldots, w(Z_j)$ and $w(Y)$ are shorted together, and all other nets are fault-free.

In order to identify all the faulty nets, we should be able to tell whether $w(Y)$ is involved in any short, which we cannot.

The sufficiency is based on the test pattern $\mathcal{T}$ described in the Lemma and the following algorithm $NGD2$ (Non-adaptive Global Diagnosis for DR2):

Algorithm. $NGD2(W, \mathcal{T})$.
Input. W is a set of nets, and $\mathcal{T}$ is a test pattern of size $|W|$.
Output. All faulty nets.

Method.
Phase 1: Apply $\mathcal{T}$ to W;
Phase 2: Report "faulty" for every net whose input is not equal to its output;
Phase 3: Report "faulty" for every net whose input is equal to the output of a "faulty" net reported in phase 2.

End of Algorithm.

To see Algorithm $NGD2(W, \mathcal{T})$ correctly identifies all faulty nets, consider a net $w(X)$ involved in a short with nets $w(Y_1), \ldots, w(Y_j)$ for some $j \geq 1$. Then the output of $w(X)$ will be $Z = X \vee Y_1 \vee \cdots \vee Y_j$. If $Z > X$, since the output of net $w(X)$ does not equal its input which is X, $w(X)$ can be detected at phase 2. If $Z = X$, from the condition of $\mathcal{T}$ in the lemma, we know $X > Y_1, \ldots, Y_j$. Therefore, nets $w(Y_1), \ldots, w(Y_j)$ will have output X and be detected at phase 2. Therefore $w(X)$ will be detected at phase 3.

On the other hand, if net w_i is identified as faulty, then either its input is not equal to its output, or its input is equal to the output of a faulty net. From the definition of $\mathcal{T}$, w_i must be involved in a short. ■

To see that the maximum independent set algorithm is not optimal for non-adaptive global diagnosis of DR2, consider a network of 5 nets. The maximum independent set algorithm uses test pattern $\{0011, 0101, 0110, 1001, 1010\}$ which is of length 4, while we use a test pattern of length 3, see Table III. However, in the following we prove the maximum independent set algorithm uses at most one more test than the optimal algorithm does.

Lemma 2: Let $\mathcal{S}$ be a collection of subsets of $\{1, 2, \ldots, t\}$, under the condition that there are no distinct $X, Y_1, \ldots, Y_j \in \mathcal{S}$, for any $j \geq 1$, such that $X = \cup_{i=1}^{j} Y_i$. Then

$$|\,\mathcal{S}\,| \leq \binom{t}{\lceil \frac{t}{2} \rceil} + \frac{2^t}{t}. \tag{2}$$

Proof: Kleitman [10] proved the above upper bound for fixed $j = 2$. In other words, he showed the bound for the maximum size of a collection of subsets of an t element set which contains no three distinct subsets X, Y, Z related by $X \cup Y = Z$. Since our requirement is stronger, the upper bound holds. ■

Lemma 2 is written in the language of the set theory. Basically, it says if the length of the test pattern is t, then we can find at most

$$\binom{t}{\lceil \frac{t}{2} \rceil} + \frac{2^t}{t}$$

TABLE III
Example non-adaptive global diagnosis.

Net	Sequential Test Vector
w_1	$X_1 = 110$
w_2	$X_2 = 101$
w_3	$X_3 = 011$
w_4	$X_4 = 100$
w_5	$X_5 = 000$

different sequential test vectors without the aliasing syndrome.

Lemma 3:

$$\binom{t}{\lceil \frac{t}{2} \rceil} + \frac{2^t}{t} < \binom{t+1}{\lceil \frac{t+1}{2} \rceil}.$$

Proof: From the Robbins' approximation of factorial [14]:

$$\sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\frac{1}{12n+1}} < n! < \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\frac{1}{12n}},$$

we have

$$\binom{2n}{n} = \frac{(2n)!}{n!n!} > \frac{\sqrt{2\pi 2n}\left(\frac{2n}{e}\right)^{2n} e^{\frac{1}{12\cdot 2n+1}}}{\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}\right)^2} > \frac{2^{2n}}{n}.$$

When t is even,

$$\binom{t+1}{\lceil \frac{t+1}{2} \rceil} = \frac{t+1}{\frac{t}{2}+1}\binom{t}{\frac{t}{2}} > \binom{t}{\frac{t}{2}} + \frac{2^t}{t}.$$

When t is odd,

$$\binom{t+1}{\lceil \frac{t+1}{2} \rceil} = 2\binom{t}{\lceil \frac{t}{2} \rceil} > \binom{t}{\lceil \frac{t}{2} \rceil} + \frac{2^t}{t}.$$

■

Theorem 2: For non-adaptive global diagnosis with DR2 (i.e., to identify all the faulty nets), the maximum independent set algorithm is not optimal, but within 1 test from optimal.

Proof: The counterexample of Table III shows the maximum independent set algorithm is not optimal. Lemma 1 and Lemma 2 give an upper bound on the number of different sequential test vectors of length t without the aliasing syndrome. Lemma 3 shows the upper bound in formula (2) is less than the size of maximum independent set of length $t+1$. Since the maximum independent set does not have the aliasing syndrome, the theorem follows. ■

To find the optimal algorithm for non-adaptive global diagnosis is difficult, since the corresponding extremal set theory problem is still open [10]. However the construction we used in Table III can be generalized as follows to obtain a slightly better method than the maximum independent set method.

In order to describe our method, we need some concepts from combinatorics [1]. Consider the *partially ordered set*, or *poset*, of all t-bit binary strings and the relation $\succ$, which is defined as follows. For any $X = x_1x_2\cdots x_t$ and $Y = y_1y_2\cdots y_t$, $X \succ Y$ if $x_i \geq y_i$ for all $i = 1, 2, \ldots, t$. X and Y are *comparable* if $X \succ Y$ or $Y \succ X$. A *chain* $\mathcal{C}$ is a set of t-bit binary strings such that elements in $\mathcal{C}$ are pairwise comparable. A chain *under* X is a chain $\mathcal{C}$ such that for any $Y \in \mathcal{C}$, $X \succ Y$. An *antichain* $\mathcal{A}$ is a set of t-bit binary strings such that elements in $\mathcal{A}$ are pairwise incomparable. A *maximum* chain or antichain is one with the maximum cardinality.

For example, consider the poset of all binary strings of length 3 in Figure 2. It is easy to see $\{111, 110, 100, 000\}$ is a maximum chain, $\{110, 101, 011\}$ is a maximum antichain, and $\{100, 000\}$ is a maximum chain under 110. The maximum independent set defined by Cheng, Lewandowski and Wu [4] is in fact a maximum antichain.

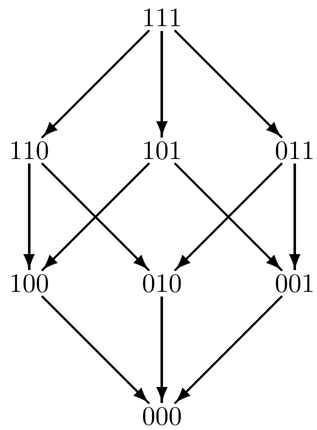

Fig. 2. Example Poset of 3-bit Binary Strings.

In our method, the test pattern $\mathcal{T}$ is the union of a maximum antichain $\mathcal{A}$ and the maximum chain $\mathcal{C}$ under X for any one element $X \in \mathcal{A}$. It is not hard to prove the so defined test pattern satisfies the condition in Lemma 1. The number of tests we need is

$$m' = \min\{i : i \text{ an integer and } \binom{i}{\lceil \frac{i}{2} \rceil} + \left\lceil \frac{i}{2} \right\rceil \geq n\}. \quad (3)$$

For example, $\{110, 101, 011\}$ is a maximum antichain, and $\{100, 000\}$ is a maximum chain under 110. Therefore our test pattern is $\{110, 101, 011, 100, 000\}$, see Table III.

C. Non-Adaptive Global Diagnosis of DR3

We know for self-diagnosis of n nets with DR3 (i.e., to identify all the connectivity), n tests are necessary because for each net w_i, we need $n-1$ bits to represent all possible shorts with other nets, and net w_i has to send 1 at least once to let other nets know their connectivity with w_i. However, this is not necessarily true for global diagnosis. In a global diagnosis, we may decide whether net w_i is in any fault using information gathered from other nets. Now we study the lower bound for non-adaptive global diagnosis with DR3, i.e., to identify all the connectivity.

Lemma 4: The necessary and sufficient condition for a test pattern $\mathcal{T}$ to identify all the connectivity for non-adaptive global diagnosis is that there are no distinct $Y_1, \ldots, Y_p, Z_1, \ldots, Z_q \in \mathcal{T}$, $p, q \geq 1$, such that

$$\vee_{i=1}^{p} Y_i = \vee_{i=1}^{q} Z_i.$$

Proof: The necessity is because otherwise we could not tell whether $w(Y_1), \ldots, w(Y_p)$ and $w(Z_1), \ldots, w(Z_q)$ are shorted into two connected components, or are all shorted together. This is also called the *confounding syndrome* [8].

The sufficiency is based on the test pattern $\mathcal{T}$ and an algorithm that reports all nets with the same output as one connected component. If there are nets $w(Y_1), w(Y_2), \ldots, w(Y_p)$, $p \geq 2$, that are shorted together, then their output must be the same. Therefore the connectivity can be detected. On the other hand, if $w(Y_1), w(Y_2), \ldots, w(Y_p)$ have the same output, by the requirement of $\mathcal{T}$, these nets must be connected together. ■

Lemma 5: Let $\mathcal{S}$ be a collection of nonempty subsets of $\{1, 2, \ldots, t\}$. If $|\mathcal{S}| \geq t+1$, then there are distinct $Y_1, \ldots, Y_p, Z_1, \ldots, Z_q \in \mathcal{S}$, $p, q \geq 1$, such that

$$\cup_{i=1}^{p} Y_i = \cup_{i=1}^{q} Z_i.$$

Proof: This is a special case of a general result by Lindström [12] and Tverberg [15]. ■

Lemma 5 is also written in the language of set theory. Basically, it says if the length of the test pattern is t, then any set of $t+1$ or more different non-zero sequential test vectors must have the confounding syndrome.

Theorem 3: For non-adaptive global diagnosis with DR3, i.e., to identify all the connected components, $n-1$ tests are necessary and sufficient, where n is the number of nets.

Proof: The necessity is from Lemma 5. If the length of the sequential test vectors is t, then we can find at most t non-zero sequential test vectors. Add $00\cdots0$, we have $t+1$ sequential test vectors. Since there are n nets, we must have $t+1 \geq n$. Therefore $t \geq n-1$.

The sufficiency is based on the walking one's method except we skip any one test. Clearly, the *walking-one's-skip-one* method satisfies the condition in Lemma 4. ■

IV. Diagnosis of Opens, Shorts and Stucks

Goel and McMahon proposed the modified counting sequence method for detection of shorts and stucks [6]. They did not consider opens explicitly since they treat opens as stuck-at-1. However, opens can be combined with shorts to form complicated connections, and therefore may not be stuck at any value. For example, we could have three nets w_1, w_2 and w_3 with the following combination of faults: the receiver of w_3 is not connected to the driver of w_3, but is shorted to the drivers of w_1 and w_2. Now, if we send the modified counting sequence 100, 010, and 110 to the drivers of the three nets respectively, then the receiver of w_3 could receive 110, as if w_3 is a good net. We give the following complete proof of the correctness of the modified counting sequence for diagnosis of opens, shorts and stucks.

Theorem 4: To non-adaptively detect opens, shorts and stucks, that is to reach DR 1, $\lceil \log(n+2) \rceil$ tests are necessary and sufficient.

Proof: In order to detect any stuck-at-0 fault, each net has to send 1 at least once. In order to detect any stuck-at-1 fault, each net has to send 0 at least once. In order to detect shorts, all the nets have to send different bit strings. Therefore, $\lceil \log(n+2) \rceil$ tests are necessary.

To prove the modified counting sequence can detect any combination of shorts, opens and stucks, it is sufficient to show we can verify for every i, whether the receiver(s) of net w_i is connected only to the driver of w_i. If a receiver of w_i is not connected to any driver, then its value will float to 0 or 1, which can be detected by the modified counting sequence. If a receiver of w_i is stuck-at-0 or stuck-at-1, it can also be detected easily. If a receiver of w_i is connected to a set of drivers of nets $w_{j_1}, w_{j_2}, \ldots, w_{j_k}$, and if $k = 1$, then it is easy to detect if w_i is connected to its own driver. If $k > 1$, then the receivers of the n nets can receive at most $n - k + 1 < n$ different output strings. Therefore we can detect the error at some receiver. ■

To adaptively diagnose opens, shorts, and stucks for single driver nets with DR3, Lien and Breuer presented an algorithm that uses $m + k$ tests [11]. Here, we show $n + 1$ tests are necessary in the worst case.

Theorem 5: To adaptively diagnose opens, shorts, and stucks of a set of n nets with DR3, $n+1$ tests are necessary and sufficient.

Proof: Just consider among all the n nets, all except one receiver r_i, are known to be stuck-at-1. Now, in order to find out which driver(s) are connected to r_i, we have to send 1 from each driver separately. In addition, we have to find out whether r_i is stuck-at-1. This gives a total of $n + 1$ tests.

On the other hand, $n+1$ tests are sufficient because $n+1$ tests are sufficient for non-adaptive diagnosis of DR3 [11]. ■

Finally, we discuss how to deal with multiple drivers. We allow either one driver to drive a net at any time, or multiple drivers to simultaneously drive a net, as long as the drivers are sending the same signal. To reach DR1, we first consider all drivers of one net as one driver and apply a length n walking one's sequence. Then we apply, in parallel within each net, a walking one's sequence respectively. It is easy to see the first step can tell us if there is any short between a receiver of net w_i and drivers of net w_j for all $i \neq j$. The second step can detect if every receiver of net w_i is connected to every receiver of net w_i, for all i. The number of tests is therefore $n + \max_{i=1}^{k} d_i$, where d_i is the number of drivers in net i.

To reach DR3, the multiple driver model is the same as the single driver model except now we should consider n as $\sum_{i=1}^{n} d_i$. This is because in order to fully diagnose the connectivity between the set of receivers and the set of drivers, any precondition on the original network is useless.

Although the number of tests for multiple drivers is the same as the number of tests for one driver, we have to keep in mind the condition that drivers of the same net can only send out the same signal at any time. Therefore, if the algorithm asks us to send 1 from driver d_i and 0 from driver d_j, where both d_i and d_j are on the same net, then we just send 1 from d_i and send nothing from d_j.

If the drivers are also receivers, then the diagnosis of DR3 for opens and shorts can be viewed as a special form of diagnosis DR3 of shorts only: Just treat each driver/receiver as one net, then the problem is in fact the interconnect diagnosis of $\sum_{i=1}^{n} d_i$ nets.

V. Conclusion

Our results are summarized in Table IV and Table V. In the tables, n is the number of nets, m and m' are defined in formula (1) and (3) respectively. For the diagnosis of shorts, we gave a divide-and-conquer algorithm for DR2 and DR3 using $\lceil \log n \rceil$ tests, an improvement of the previous two-step $\lceil \log n \rceil + k$. We also proved matching lower bounds for non-adaptive diagnosis of DR2 and DR3. For the diagnosis of opens, shorts, and stucks, we proved $\lceil \log(n+2) \rceil$ tests are necessary and sufficient for DR1, and $n+1$ tests are necessary and sufficient for DR3. We leave as an open problem to find an optimal adaptive algorithm to identify all faulty nets of shorts, stucks and opens.

One conclusion drawn from our results is that for non-adaptive diagnosis, global information provides little help. We show global diagnosis is better than self diagnosis by at most 1 test. For adaptive diagnosis, global information does help, and the test can be sped up substantially.

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TABLE IV
Current best algorithms for shorts.

Diagnostic Resolution	**Adaptive**	**Global**	**Number of Tests**	**Optimal**	**Authors**
1	no	no	$\lceil \log n \rceil$	yes	Kautz [9]
2	no	no	m	yes	Cheng, Lewandowski and Wu[4]
	no	yes	m'	within 1	Shi and Fuchs
	yes	yes	$\lceil \log n \rceil$	yes	Shi and Fuchs
3	no	no	n	yes	Hassan, Rajski and Agarwal[7]
	no	yes	$n-1$	yes	Shi and Fuchs
	yes	yes	$\lceil \log n \rceil$	yes	Shi and Fuchs

TABLE V
Current best algorithms for shorts, opens and stucks

Diagnostic Resolution	**Adaptive**	**Number of Tests**	**Optimal**	**Authors**
1	no	$\lceil \log(n+2) \rceil$	yes	Goel and McMahon [6] Shi and Fuchs
2	no	m	yes	Cheng, Lewandowski and Wu [4]
	yes	m	?	
3	no	$n+1$	yes	Lien and Breuer [11]
	yes	$n+1$	yes	Shi and Fuchs